

Superconductivity notes (Andrew Boothroyd)

Resistance

Slide 1

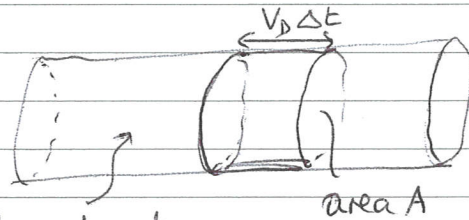
- Pinball model of resistance
- Ohm's Law
- Resistivity

— Calculation of drift velocity and current density —

Two relevant electron velocities:

1. Instantaneous velocity $v \sim 10^5 - 10^6 \text{ m s}^{-1}$
2. Drift velocity v_D

Charge that flows in time Δt
 $= neAv_D\Delta t$



number density
of electrons = n

$$\Rightarrow I = neAv_D$$

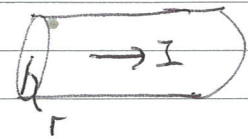
Current per unit area [eliminates shape of wire]

$$j = \frac{I}{A}$$

$$= nev_D \quad \text{--- (1)}$$

Estimate v_D :

Take $r = 1 \text{ mm}$, $I = 0.5 \text{ A}$, $n = 10^{28} \text{ m}^{-3}$



$$\begin{aligned} \rightarrow j &= \frac{I}{\pi r^2} \\ &= \frac{0.5}{\pi (10^{-3})^2} \\ &\approx 1.6 \times 10^5 \text{ A m}^{-2} \end{aligned}$$

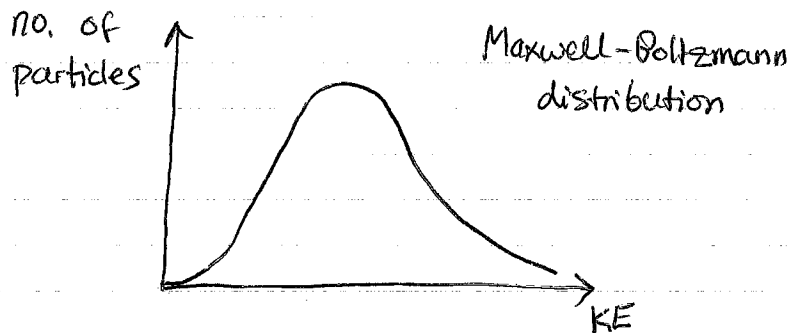
$$\begin{aligned} \text{(1)} \rightarrow v_D &= \frac{j}{ne} \\ &= \frac{1.6 \times 10^5}{10^{28} \times 1.6 \times 10^{-19}} \text{ m s}^{-1} \\ &\approx 10^{-4} \text{ m s}^{-1} \end{aligned}$$

→ slide 1 Resistivity varies with temperature

Critical temperature T_c

Classical gas of particles at temperature T :

$$\langle KE \rangle = \frac{3}{2} k_B T$$



Critical current density, j_c

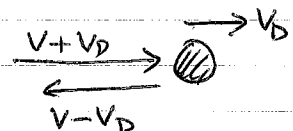
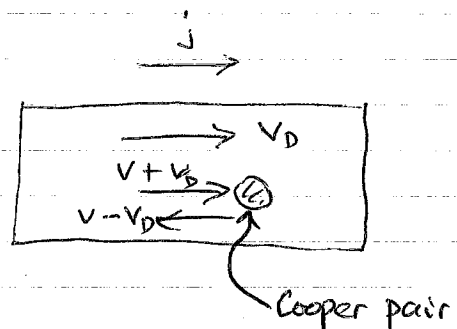
Speed of electrons in absence of current = v

Cooper pair can be destroyed by collision with another electron

Consider a head-on collision:

Change in KE due to collision is

$$\begin{aligned} \Delta KE &= \frac{1}{2} m (v + v_D)^2 - \frac{1}{2} m (v - v_D)^2 \\ &= \frac{1}{2} m [(v^2 + 2vv_D + v_D^2) - (v^2 - 2vv_D + v_D^2)] \\ &= 2mvv_D \end{aligned}$$



Cooper pair is destroyed when $2\Delta \approx \Delta KE$

i.e. $2\Delta \approx 2mvv_D$

Since $j = nev_D$ the critical current density is

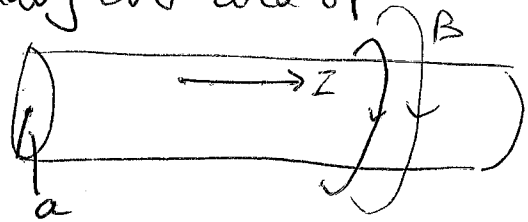
$$\begin{aligned} j_c &= \frac{ne\Delta}{mv} \\ &\approx \frac{nek_B T_c}{mv} \end{aligned}$$

Take $n = 10^{28} \text{ m}^{-3}$, $v = 10^6 \text{ ms}^{-1}$, $T_c = 10 \text{ K}$

$$\rightarrow \underline{j_c \approx 10^{11} \text{ A m}^{-2}}$$

Critical magnetic field

Assume current flows uniformly over area of wire (not true).



$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$j = \frac{I}{\pi a^2}$$

→ field at surface ($r=a$) is

$$B(a) = \frac{\mu_0 j a}{2}$$

Critical field is when $j = j_c \sim 10^{11} \text{ Am}^{-2}$.

Take $a = 0.5 \times 10^{-3} \text{ m}$

$$\rightarrow B_c = \frac{4\pi \times 10^{-7} \times 10^{11} \times 0.5 \times 10^{-3}}{2}$$

$$\approx \underline{\underline{30 \text{ Tesla}}}$$