Superconductivity notes (Andrew Boothroyd)

Resistance

Slide 1

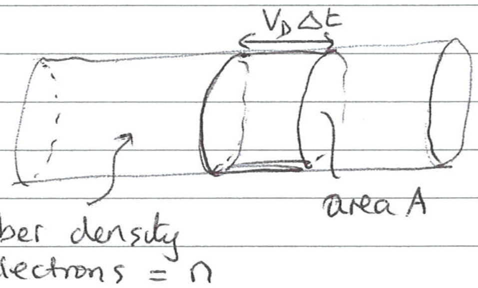
- Pinball model of resistance
- Ohm's law
- Resistivity

— Calculation of drift velocity and current density —

Two relevent electron velocities :

1. Instantaneous velocity $v \sim 10^5 - 10^6\ \mathrm{ms^{-1}}$

2. Drift velocity V_D



Charge that flows in time Δt

$= neAV_D\Delta t$

$\Rightarrow I = neAV_D$

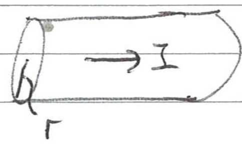
Current per unit area [eliminates shape of wire]

$j = \frac{I}{A}$

$= nev_D$ —— ①

Estimate V_D:

Take $r = 1\,\mathrm{mm}$, $I = 0.5\,\mathrm{A}$, $n = 10^{28}\,\mathrm{m^{-3}}$



$\rightarrow j = \frac{I}{\pi r^2}$

$= \frac{0.5}{\pi (10^{-3})^2}$

$\simeq 1.6 \times 10^5\ \mathrm{A\,m^{-2}}$

① $\rightarrow V_D = \frac{j}{ne}$

$= \frac{1.6\times 10^5}{10^{28} \times 1.6 \times 10^{-19}}\ \mathrm{ms^{-1}}$

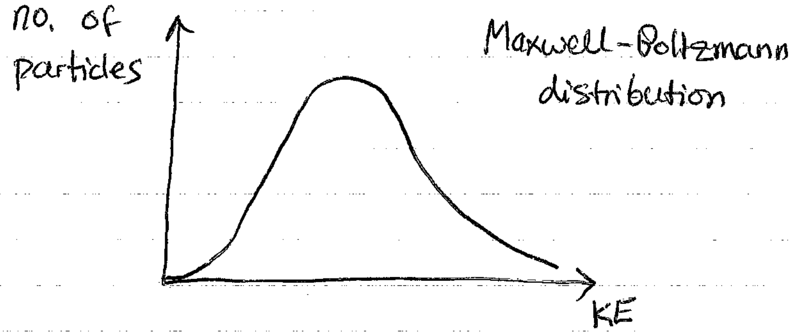
$\simeq 10^{-4}\ \mathrm{ms^{-1}}$

→ slide 1 Resistivity varies with temperature

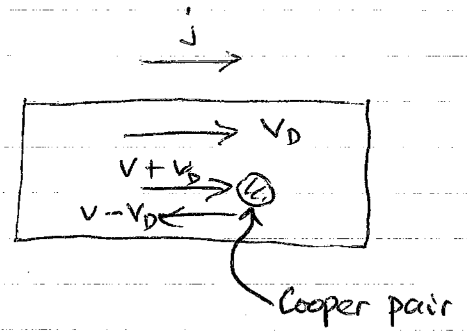
Critical temperature T_c

Classical gas of particles at temperature T:

$$\langle KE \rangle = \frac{3}{2} k_B T$$



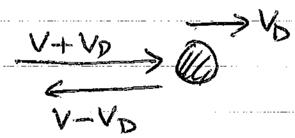
Critical current density, j_c



Speed of electrons in absence of current $= V$

Cooper pair can be destroyed by collision with another electron

Consider a head-on collision:



Change in KE due to collision is

$$\begin{aligned} \Delta KE &= \tfrac{1}{2}m(V+V_D)^2 - \tfrac{1}{2}m(V-V_D)^2 \\ &= \tfrac{1}{2}m\left[(V^2 + 2VV_D + V_D^2) - (V^2 - 2VV_D + V_D^2)\right] \\ &= 2mVV_D \end{aligned}$$

Cooper pair is destroyed when $2\Delta \simeq \Delta KE$

i.e. $2\Delta \simeq 2mVV_D$

Since $j = neV_D$ the critical current density is

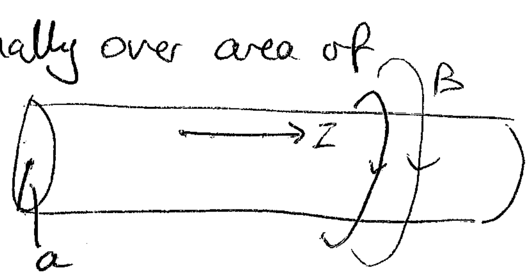
$$\begin{aligned} j_c &= \frac{ne\Delta}{mV} \\ &\simeq \frac{nek_BT_c}{mV} \end{aligned}$$

Take $n = 10^{28}\,m^{-3}$, $V = 10^6\,ms^{-1}$, $T_c = 10\,K$

$$\longrightarrow j_c \simeq 10^{11}\,Am^{-2}$$

Critical magnetic field

Assume current flows uniformally over area of wire (not true).



$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$j = \frac{I}{\pi a^2}$$

→ field at surface ($r=a$) is

$$B(a) = \frac{\mu_0 j a}{2}$$

Critical field is when $j = j_c \sim 10^{11}\,\mathrm{Am^{-2}}$.
Take $a = 0.5\times10^{-3}\,\mathrm{m}$

$$\rightarrow \quad B_c = \frac{4\pi\times10^{-7}\times10^{11}\times0.5\times10^{-3}}{2}$$

$$\simeq \underline{30 \text{ Tesla}}$$